

**RAN-2203000206020032**
**T. Y. B. Sc. Mathematics (Sem.-VI) Examination March - 2026**
**MTH-602- Linear Algebra - II (LOD)**
**Time: 2 Hours ]**
**[ Total Marks: 50**

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| <b>સૂચના : / Instructions</b><br>(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.<br>Fill up strictly the above details on your answer book<br>(2) All questions are compulsory.<br>(3) Figures to the right indicate marks of the questions.<br>(4) Follow usual notations. | Seat No.:<br><div style="display: flex; justify-content: space-around; width: 100%;"> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> <div style="border: 1px solid black; width: 20px; height: 20px;"></div> </div> <div style="border: 1px solid black; width: 100%; height: 80px; margin-top: 10px;"></div> <div style="text-align: center; margin-top: 5px;">Student's Signature</div> |
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**Q. 1. Answer the following questions (Any Five):**
**10**

1. Prove that Identity map is linear in a vector space  $V_2$ .
2. Let  $T : V_3 \rightarrow V_2$  be a linear map define by  $T(x_1, x_2, x_3) = (x_1, x_1 + x_2)$ .  
Is  $T$  a 1-1 linear map? Justify your answer.
3. Find the linear map associate with a matrix  $\begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$  relative to both standard basis.
4. Let  $T : V_3 \rightarrow V_2$  define by  $T(e_1) = (2, 1)$ ,  $T(e_2) = (0, 1)$ ,  $T(e_3) = (1, \frac{1}{2})$ . Find  $\dim R(T)$ .
5. Find the rank of the matrix  $\begin{bmatrix} 2 & 4 & 2 \\ 5 & 3 & -2 \end{bmatrix}$ .
6. Prove that A linear transformation  $T : V_2 \rightarrow V_2$  is one-one then  $n(t) = 0$ .
7. State two difference between Euclidean space and Unitary space in an Inner product space.
8. Give an example of an Orthonormal set which is L.I in an inner product Space  $V$ .

**Q. 2. Answer the following (Any two).**
**10**

1. Let  $T : U \rightarrow V$  be a linear map. Prove that a linear map  $T : U \rightarrow V$  is one –one if and only if the null space is itself a zero of the space.
2. If  $v_1, v_2, v_3, \dots, v_n$  are  $n$  linearly independent vectors of  $R(T)$  and  
if  $u_1, u_2, u_3, \dots, u_n$  are  $n$  vectors of  $U$  with condition that  $T(u_i) = v_i$ ,  
 $\forall i = 1$  to  $n$  for a linear map  $T : U \rightarrow V$  then prove that  $u_1, u_2, u_3, \dots, u_n$  are L. I  
vectors of  $U$ .
3. Obtain the general rule of the linear transformation  $T : V_2 \rightarrow V_2$  defined by  
 $T(4, 1) = (1, 2, 3)$ ,  $T(2, -1) = (6, -3)$ .

- Q. 3. Answer the following (Any two). 10**
1. Prove that the linear transformation  $T: V_3 \rightarrow V_3$  such that  $T(e_1) = e_1 - e_2$ ,  $T(e_2) = 2e_2 + e_3$  and  $T(e_3) = e_1 + e_2 + e_3$  is neither one-to-one nor onto.
  2. Find the inverse linear transformation of the linear transformation  $T$  such that  $T: V_3 \rightarrow V_3$ ;  $T(e_1) = e_1 + e_2$ ,  $T(e_2) = e_2 + e_3$ ,  $T(e_3) = e_1 + e_2 + e_3$  if it exists.
  3. Find Range, Rank, Kernel and Nullity for a linear transformation  $T: V_3 \rightarrow V_2$ , such that  $T(e_1) = (2, 1)$ ,  $T(e_2) = (3, 2)$ ,  $T(e_3) = (1, 1)$ .
- Q. 4. Answer the following (Any two). 10**
1. Find the matrix  $(T; B_1, B_2)$  associated with a linear transformation  $T: V_2 \rightarrow V_2$  defined by  $T(x, y) = (x, -y)$  relative to basis  $B_1 = \{(1, 1), (1, 0)\}$  and  $B_2 = \{(2, 3), (4, 5)\}$ .
  2. Find the Range, Rank, Kernel and Nullity of a matrix  $\begin{bmatrix} 5 & 1 & 6 & 0 \\ 0 & 3 & 3 & 0 \\ 4 & -4 & 0 & 0 \end{bmatrix}$
  3. Find the Linear transformation  $T$  associated with a matrix  $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$  relative to basis  $B_1 = \{(1, -1, 1), (1, 2, 0), (0, -1, 0)\}$  and  $B_2 = \{(1, 1), (2, -1)\}$ .
- Q. 5. Answer the following (Any two). 10**
1. In an Inner Product space  $V$ , Prove that  $\|u + v\| \leq \|u\| + \|v\|$ ,  $\forall u, v \in V$  and  $\alpha$  a scalar
  2. Define Norm of a vector and In an Inner Product space  $V$ , Prove that  $\|\alpha v\| = |\alpha| \|v\|$ ,  $\forall v \in V$  and  $\alpha$  a scalar.
  3. Orthonormalized the L.I set  $\{(1, 0, 0), (1, 1, 1), (1, 2, 3)\}$  by Gram Schmidt's process.